

Using the unit labels to your advantage

Name _____

No one method is best for solving every type of problem. One good approach is a method called dimensional analysis. It is a way to approach, analyze, and solve problems using the units of the measurements to help guide you to the correct answer. Using the units can help to check your work and avoid mechanical errors.

Consider the following problem:

My dog is awake 10 hours per day and drinks 2.0 cups of water every 3.0 hours.

Calculate how many gallons of water she will drink in her 14 year life span.

In order to set up this problem using dimensional analysis, you can think of it as a “trip”. The two things that you know about this “trip” is where you start and where you want to end up. Conversion factors allow you to get from here to there. First think about the units that should be on the answer. In this case: gallons/lifetime. This lets you know that you need a volume in the numerator and a time in the denominator.

Conversion factors relate a quantity in a certain unit to a quantity (often 1) of another unit
such as 60 min per hour 60 min / 1 hr.

So for the problem above you need to determine the appropriate conversion factors that will allow you to take the “trip”.

Some conversion factors come from the problem:

$$\frac{10\text{hours}}{1\text{day}} \quad \text{in the dog's awake day, and} \quad \frac{14\text{years}}{1\text{lifetime}}$$

Some conversion factors can be looked up, perhaps you'll need to look some up on-line when solving practice problems.

$$\frac{4\text{cups}}{1\text{quart}} \quad \frac{4\text{quarts}}{1\text{gallon}} \quad \frac{24\text{hours}}{1\text{day}} \quad \text{but only} \quad \frac{10\text{hours}}{1\text{dogday}} \quad \frac{365\text{days}}{1\text{year}}$$

Remember that every conversion factor can be used as a multiplier while solving the problem.

The conversion factors can be written as they are written above, or in their inverse form, i.e. $\frac{4\text{cups}}{1\text{quart}}$ or $\frac{1\text{quart}}{4\text{cups}}$

The units need to be cancelled will determine which units should be in the denominator, and which units should be in the numerator.

Solve:

$$\frac{2\text{cups}}{3.0\text{hrs}} \frac{1\text{quart}}{4\text{cups}} \frac{1\text{gallon}}{4\text{quarts}} \frac{10\text{hrs}}{1\text{dogday}} \frac{365\text{days}}{1\text{yr}} \frac{14\text{yrs}}{1\text{lifetime}} = 2100 \frac{\text{gallons}}{\text{lifetime}}$$

Notice that all the units cancel out except for gallons and lifetime.

A word about significant figures.

Be aware of conversion factors that are not a result of measurements but instead are exact definitions. All of the conversion factors given above are exact definitions. Exact relationships are considered to have no uncertainty, and therefore unlimited number of significant figures. They do not determine the number of significant figures in a calculation any other measurements in the calculation will be the determining factor in your rounding off procedure.

Whenever your problem involves a conversion between units within the metric system, you can simply shift the decimal point before or after the problem, or you can use conversion factors to convert between units, i.e. 1000 ml/1 L

Evidence and Testimonials to provide evidence as to why you need Dimensional Analysis

The evidence: A ten year study at Northampton High School from 2005–2015 show that 100% of high school students who do not use and understand dimensional analysis are seriously insecure by their junior year. Damage done from this deprivation in the first two years of high school is probably permanent and cannot be overcome by learning the method later in life. We recommend mastering this skill before the end of your sophomore year.

83% of the students surveyed admitted that they enjoyed solving problems with Dimensional Analysis in order to impress and confuse their parents at dinner time. Of the remaining 17%, 11% never ate dinner together with their parents, and the remaining 6% never talked to their parents.

"I was a Northampton High School student who dozed off while Mr Daudelin taught us dimensional analysis in physics. I never quite got the hang of it. It irritated me... all of those fractions. I never really liked fractions. Although my grades had been pretty high, I got a D in physics and subsequently dropped out of school in the first quarter of my junior year. It was not long before I started on drugs, and then turned to crime to support my drug habit. I have recently learned dimensional analysis and realize how simply FLM could have solved all of my problems. Alas, it is too late. I won't get out of prison until 2024 and even then, my self image is permanently damaged. I attribute all of my problems to my unwillingness to learn dimensional analysis."
Jane NHS '08

"I thought I knew everything and that sports were the only thing that mattered in high school. When Ms. Patch taught our class dimensional analysis, I didn't care about it at all. I was making plans for the weekend with my girlfriend who loved me because I was a running back and not because of my chemistry prowess. While other kids were home solving dimensional analysis problems, I was practicing making end sweeps. Then one day I was hit hard. Splat. My knee was gone. I was despondent. My girlfriend deserted me. My parents, who used to brag about my football stats, started getting on my case about grades. I decided to throw myself into my school work. But I couldn't understand anything. I would get wrong answers all of the time. I now realize that my failure in school came from never having learned dimensional analysis. Alas, I thought everyone else was smarter. After the constant humiliation of failing I finally gave up. I am worthless. I have no friends, no skills, no interests. I have now learned dimensional analysis, but it seems too late."
Bill NHS '11

"I was at home, sick with the flu when Ms Biggs taught my class about Dimensional Analysis. Despite opportunities given to me to make up the assignments that I had missed, I chose to not do them. I thought that my mathematical abilities were already sufficient. How wrong I was! It's been 8 years since I took that class--Now I spend my afternoons panhandling at traffic lights, hoping for passersby to give me spare change. If I'm lucky enough to come up with a few bucks after a day's work at the light, I'm still not sure if my hourly rate makes cents."
Dave NHS '07